YEAR 13 – MATHMATICS

WEEK 1 – DAY 1

STRAND 1 – COMPLEX NUMBERS

Simplify $[2(\cos 70° + i \sin 70°)]^9$.

Give your answer in **rectangular form**.

Simplify $\dfrac{2 + 3i}{-3 + 2i}$

Find the values of x and y such that

$x + iy = \sqrt{4} - \sqrt{-9} + \sqrt{-16}$

Solve the equation $z^2 = -32 + 32\sqrt{3}i$

Leave your answers in **polar form**.

WEEK 1 – DAY 2

STRAND 2 – VECTORS

Find the **unit vector** that has

the same direction as $\underset{\sim}{v} = 2\boldsymbol{i} - \boldsymbol{j} - 2\boldsymbol{k}$

Find **parametric equations** of the line passing through the points $(3, 4, -1)$ and $(9, 0, 7)$

Let $\underset{\sim}{a} = \begin{pmatrix} 4 \\ k \\ 7 \end{pmatrix}$ and $\underset{\sim}{b} = \begin{pmatrix} 4 \\ 1 \\ -2 \end{pmatrix}$.

Find the value of k if $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are **orthogonal**.

Let point A = $(-3, 4, 7)$ and point B = $(5, 20, -9)$

Determine the **coordinates** of point P on

the line AB given that AB = 4AP

WEEK 1 – DAY 3

STRAND 3 – FUNCTIONS

Sketch the graph given below:

$$y = -\frac{1}{4}(x - 1)^3(x + 1)^2(x - 4)$$

Sketch the graph given below:

$$f(x) = \frac{x^2 + x}{-2x + 2}$$

Let $f(x) = 7 - x^2$ and $g(x) = \sqrt{x - 1}$

Find $fog(x)$ and state its range.

WEEK 1 – DAY 4

STRAND 4 – TRIGONOMETRY

Let $\cos\theta - \sin\theta = R\cos(\theta - \alpha)$

Find the values of R and α. Hence or otherwise sketch the graph $\cos\theta - \sin\theta$.

Solve $\frac{1}{\sin\theta} + 2\sin\theta = -3$ for $0° \le \theta \le 360°$

Prove the following identity

$$\sin^{-1}x + \cos^{-1}x = \frac{1}{2}\pi$$

WEEK 1 – DAY 5

STRAND 1 – COMPLEX NUMBERS

Evaluate $\sqrt{-100}$

$u = 3(\cos 90° + i\sin 90°)$
$v = 5(\cos 180° + i\sin 180°)$

Find uv

Express $\frac{13}{3+2i}$ in the form $a+bi$

Solve the equation $z^2 = 196i$

Leave your answers in **polar form**.

WEEK 2 – DAY 1

STRAND 2 – VECTORS

Consider the following **three dimensional** vectors:

$$\underline{a} = 4\boldsymbol{i} - 2\boldsymbol{j} - 4\boldsymbol{k}$$
$$\underline{b} = -2\boldsymbol{i} + 3\boldsymbol{j} - 6\boldsymbol{k}$$

(a) Find $|\underline{a}|$

(b) Find $|\underline{b}|$

(c) Determine the **dot product** of $\underline{a}$ and $\underline{b}$

(d) Hence, calculate the **angle** between $\underline{a}$ and $\underline{b}$

The equation of a line in **symmetric form** is

$$\frac{x-1}{4} = \frac{y-2}{8} = \frac{z+3}{12}$$

Give the coordinates of **a point** which lies on this line.

Let A be the point (12, 3, 4) and let B be the point $(-6, 12, -5)$. Find the **coordinates** of point P on the line AB given that

$$\frac{AP}{PB} = \frac{2}{7}$$

WEEK 2 – DAY 3

STRAND 3 – FUNCTIONS

$f(x)=x^2$ and $g(x)=\sqrt{x-4}$

Find an expression for $f\,o\,g\,(x)$.

State the **domain** of $f\,o\,g\,(x)$.

Sketch the graph of:

$y=(x-1)(x+1)^3(x-3)^2$

Sketch the graph of:

$f(x)=\dfrac{(x+1)(x+4)}{(x-2)(x+2)}$

WEEK 2 – DAY 4

STRAND 4 – TRIGONOMETRY

$y=6\sin\theta+8\cos\theta$

Express the above function as:

$r\cos(\theta-\alpha)$

Find the coordinates of the minimum point on this function, for $0° \leq \theta \leq 360°$.

Prove that:

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

Solve $\cos^2\theta=\dfrac{3}{4}$ for $0° \leq \theta \leq 360°$

WEEK 2 – DAY 5

STRAND 1 – COMPLEX NUMBERS

A complex number is given as $z=2+3i$

Find

(a) $Re(z)$, the **real part** of z

(b) $Im(z)$, the **imaginary part** of z

(c) $\bar{z}$, the **conjugate** of z

(d) $z+\bar{z}$

Use the **quadratic formula** $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

to solve $x^2-10x+26=0$

A complex number is given as $w=1+\sqrt{3}i$

Find Arg(w), the **argument** of w.

Find $|w|$, the **modulus** of w.

Convert w into **polar form**.

Hence, evaluate w^4

$z^3=216(\cos 60°+i\sin 60°)$

Leave your answers in **polar form**.

WEEK 3 – DAY 1

STRAND 2 – VECTORS

$$\underset{\sim}{a}=\begin{pmatrix}2\\-4\\-4\end{pmatrix} \text{ and } \underset{\sim}{b}=\begin{pmatrix}2\\-1\\-2\end{pmatrix}$$

(i) Find $|\underset{\sim}{a}|$

(ii) Find $|\underset{\sim}{b}|$

(iii) Determine the **dot product** of $\underset{\sim}{a}$ and $\underset{\sim}{b}$

(iv) Hence, calculate the **angle** between $\underset{\sim}{a}$ and $\underset{\sim}{b}$

$$\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1\\2\\3\end{pmatrix}+t\begin{pmatrix}2\\4\\6\end{pmatrix}$$

Express this equation in **symmetric form**

If A is the point $(-2, 5, 12)$ and B is the point $(13, -5, -18)$, find the **coordinates** of point P on the line AB given that AP : PB = 1 : 4

WEEK 3 – DAY 2

STRAND 3 – FUNCTIONS

$$f(x)=(x-3)^2 \text{ and } g(x)=x+3$$

Find $fog(x)$ and state its domain.

Sketch the following graphs:

$$y=(x+2)^4(x-1)^3.$$

$$f(x)=\frac{x-6}{(x-2)(x+3)}$$

WEEK 3 – DAY 3

STRAND 4 – TRIGONOMETRY

Prove that:

$$\frac{\sin 6\theta+\sin 4\theta}{\cos 6\theta+\cos 4\theta}=\tan 5\theta$$

$$y=5\sin\theta+12\cos\theta$$

Express the above function as:

$$r\cos(\theta-\alpha)$$

Find the coordinates of the maximum point on this function, for $0° \leq \theta \leq 360°$.

Solve $(2\sin\theta+1)(\sin\theta-1)=0$

for $0° \leq \theta \leq 360°$

WEEK 3 – DAY 4

STRAND 1 – COMPLEX NUMBERS

Express:

$$\frac{5}{2+i} \text{ in the form } a+bi$$

Solve:

$x^2 - 2x = -5$

$w = \sqrt{12} + 2i$

Find Arg(w), the **argument** of w.

Find $|w|$, the **modulus** of w.

Convert w into **polar form**.

Hence, evaluate w^3

Solve:

$z^2 = 64(\cos 90° + i \sin 90°)$

Express your answers in **rectangular form**.

WEEK 3 – DAY 5

STRAND 2 – VECTORS

$\underset{\sim}{a} = 6\boldsymbol{i} - 2\boldsymbol{j} + 4\boldsymbol{k}$ and $\underset{\sim}{b} = -9\boldsymbol{i} + 3\boldsymbol{j} - 6\boldsymbol{k}$

Find $|\underset{\sim}{a}|$

Find $|\underset{\sim}{b}|$

Determine the **dot product** of $\underset{\sim}{a}$ and $\underset{\sim}{b}$

Hence, calculate the **angle** between $\underset{\sim}{a}$ and $\underset{\sim}{b}$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + t\begin{pmatrix} 4 \\ -5 \\ 6 \end{pmatrix}$$

Give the coordinates of a **point** that lies on this line.

Give a **direction vector** of this line.

If A is the point (12, 3, 4) and B is the point $(-6, 12, -5)$, find the **coordinates** of point P on the line AB given that $\frac{AP}{AB} = \frac{2}{9}$

WEEK 4 – DAY 1

STRAND 3 – FUNCTIONS

$f(x) = x^2$ and $g(x) = \sqrt{x-5}$

State the domain of $g(x)$.

Sketch the graphs of:

$y = (x+2)^3(1-x)^2$

$f(x) = \dfrac{(x-3)(x+1)}{(x-1)(x+2)}$

WEEK 4 – DAY 2

STRAND 4 – TRIGONOMETRY

Prove that:

$$\tan 2\theta + \tan 5\theta = \frac{\sin 7\theta}{\cos 2\theta \cos 5\theta}$$

$$y = \sqrt{2}\sin\theta + \sqrt{2}\cos\theta$$

Express the above function as:

$r\cos(\theta - \alpha)$

Find the coordinates of the minimum point on this function, for $0° \leq \theta \leq 360°$.

Solve:

$$\cos\theta(2\cos\theta + \sqrt{3}) = 0 \text{ for } 0° \leq \theta \leq 180°$$

WEEK 4 – DAY 3

STRAND 1 – COMPLEX NUMBERS

$v = 2 + 3i$ and $w = 5 + 4i$

(a) $|v|$

(b) $v + w$

(c) $\overline{w}$

Solve:

$$x^2 - 6x + 25 = 0$$

Express $\frac{2}{1-i}$ in the form $a + bi$.

$w = 1 + \sqrt{3}i$.

Write w in **polar form**.

Use **De Moivre's Theorem** to find w^5

Solve the equation $z^3 = -8i$.

WEEK 4 – DAY 4

STRAND 2 – VECTORS

$$\underset{\sim}{a} = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} \text{ and } \underset{\sim}{b} = \begin{pmatrix} 6 \\ 2 \\ 1 \end{pmatrix}$$

Find $\underset{\sim}{b} - \underset{\sim}{a}$.

Determine the scalar product of the vectors

Find the angle between the vectors

Find the equation of a line passing through the point $(1, 2, -3)$ and parallel to the vector $\begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix}$ in:

parametric form.

symmetric form .

If P is the point $(1, -1, 2)$ and R is the point $(4, 9, -3)$, find the coordinates of a point Q on the line PR given that PQ : QR = 2 : 3.

WEEK 4 – DAY 5

STRAND 3 – FUNCTIONS

$f : x \to x^2$ and $g : x \to x - 3$.

Find:

$f \circ g\,(x)$

$g^{-1}(x)$

Sketch the graphs of:

$y = (x-1)^3(x+2)^2$

$f(x) = \dfrac{x+3}{(x-3)(x+2)}$.

WEEK 5 – DAY 1

STRAND 4 – TRIGONOMETRY

Prove that:

$$\frac{2\tan\theta}{1+\tan^2\theta} = \sin 2\theta.$$

$y = 6\sin\theta + 8\cos\theta$.

Express the above function as:

$r\sin(\theta + \alpha)$

Find the coordinates of the minimum and maximum points on this function,

for $0° \le \theta \le 360°$.

Solve:

$\tan^2\theta = \tan\theta$ for $0° \le \theta \le 180°$.

WEEK 5 – DAY 2

STRAND 1 – COMPLEX NUMBERS

$Z = 5\ \text{CIS}\ \dfrac{\pi}{2}$:

Find: $|Z|$ and $\arg(Z)$.

Solve $x^2 - 4x + 7 = 0$

Express $\frac{1}{2 - 3i}$ in the form $a + bi$

$W = -\sqrt{3} - i$:

Write W in polar form.

Use De Moivre's Theorem to find W^3

Express the answer in **rectangular form.**

Solve the equation $z^3 - i = 0$.

Express the answer in **rectangular form.**

WEEK 5 – DAY 3

STRAND 2 – VECTORS

$\underset{\sim}{\mathbf{u}} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ and $\underset{\sim}{\mathbf{v}} = \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix}$

Express the vector VU in terms of the unit vectors $\underset{\sim}{\mathbf{i}}$, $\underset{\sim}{\mathbf{j}}$ and $\underset{\sim}{\mathbf{k}}$.

Find the angle between $\underset{\sim}{\mathbf{u}}$ and $\underset{\sim}{\mathbf{v}}$.

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix} + t \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$

Give a directional vector of this line.

Give the coordinates of a point that lies on this line.

Write the Cartesian equation of the line passing through the point (2, 6, –1) in the direction of $\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$

If A is the point (7, 4, –2) B is the point (1, 2, –10), find the coordinates of point P on the line AB given that AB = 6AP.

WEEK 5 – DAY 4

STRAND 3 – FUNCTIONS

$f : x \to x^2 + 1$ and $g : x \to 4x - 2$

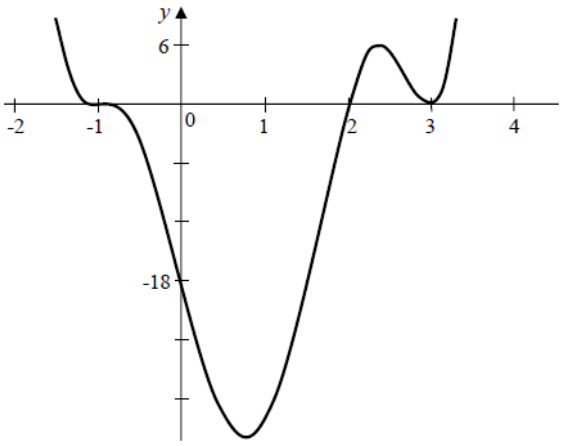
Find: a) $f - g(x)$

b) Domain of $f - g(x)$ c) $fog(x)$

Sketch the graph of:

$$g(x) = \frac{x^2 - x - 2}{x + 3}$$

Write the equation of the function below:



WEEK 5 – DAY 5

STRAND 4 – TRIGONOMETRY

Prove that:

$$\frac{1 - \cos 2x}{1 - \sin^2 x} = 2\tan^2 x$$

Express $\sin\frac{\pi}{4}\sin\frac{\pi}{12}$ as a sum.

Simplify your answer.

$y = \cos\theta + \sin\theta$:

Write in the form $y = r\sin(\theta + \alpha)$

Sketch the graph of y for $0 \leq \theta \leq 2\pi$.

WEEK 6 – DAY 1

STRAND 1 – COMPLEX NUMBERS

$V = 1 - 5i$ and $W = -1 + 3i$

Find:

$$\frac{V}{W}$$

$$\overline{VW}$$

shade the region where $-1 < \mathrm{Im}(z) \leq 2$

Write $Z = \frac{1 - 3i}{1 + 3i}$ in the polar form.

Find $Z = \left(\frac{1 - 3i}{1 + 3i}\right)^2$

in **rectangular form**.

Solve the equation $Z^3 = -27i$.

Give your answer in **rectangular form**.

WEEK 6 – DAY 2

STRAND 2 – VECTORS

$\underset{\sim}{\mathbf{a}} = \underset{\sim}{\mathbf{i}} + 2\underset{\sim}{\mathbf{j}} + 3\underset{\sim}{\mathbf{k}}$ and $\underset{\sim}{\mathbf{b}} = \underset{\sim}{\mathbf{i}} + 3\underset{\sim}{\mathbf{j}} - \underset{\sim}{\mathbf{k}}$

$\underset{\sim}{\mathbf{a}} - 2\underset{\sim}{\mathbf{b}}$

the angle between vectors $\underset{\sim}{\mathbf{a}}$ and $\underset{\sim}{\mathbf{b}}$

Find the vector equation of the line passing through $(-1, 2, 4)$ and parallel to $(2, -2, -4)$ in:

parametric form

symmetric form

If A is the point $(-1, 4, 10)$ and B is the point $(12, -1, -13)$,

find the coordinates of point P on the line AB given that AP : PB = 1 : 3.

WEEK 6 – DAY 3

STRAND 3 – FUNCTIONS

$f : x \to \sqrt{x - 1}$ and $g : x \to x^2$

Find:

$fog\,(x)$

$g^{-1}(x)$

Sketch the graph of:

$$g(x) = \frac{x - 3}{x^2 - 4}$$

WEEK 6 – DAY 4

STRAND 4 – TRIGONOMETRY

Prove that:

$$\frac{1}{1 + \text{Sin}^2\theta} + \frac{1}{1 + \text{Csc}^2\theta} = 1$$

Sketch the graph of $y = \sin^{-1} x$ for $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$.

$y = 4\cos\theta + 3\sin\theta$:

Write in the form $y = r\cos(\theta + \alpha)$

Find the coordinates of the minimum and maximum points on this function, for $0° \leq \theta \leq 360°$.

WEEK 6 – DAY 5

$V = 3 + 5i$ and $W = -1 + i$

Find:

$\overline{V}$

$|V - W|$

If $\sin x = \frac{3}{5}$ and $\sin y = \frac{5}{13}$,

find the exact value of $\cos(x - y)$.